C. When Einstein did Solid State Physics
- Heat capacity of insulators

D. Photoelectric Effect – Implication and Modern applications

E. Compton Scattering – Particle Nature of Light confirmed

C. When Einstein did Solid State Physics

Heat Capacity of Insulators $C(\overset{\text{temp.}}{T})$

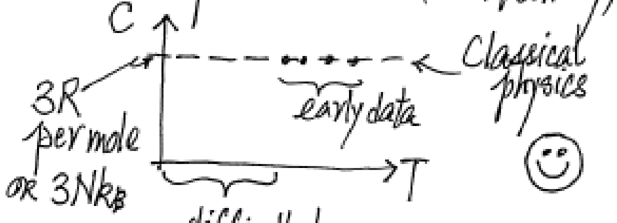
- Classical Physics predicts:

$$C(T) = 3\underset{\uparrow\ \#\text{ atoms}}{N}k_B \leftarrow \text{Boltzmann constant}$$

(independent of temperature)

- Experiments
  - C(T) drops as T drops
  - [Another failure of classical Physics]

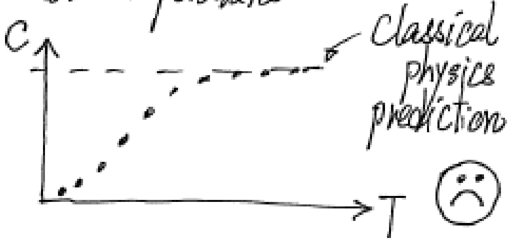
- Once Upon a time... (late 19th century)



$R = 8.314\,\text{J}\cdot\text{mol}^{-1}\cdot\text{K}^{-1}$

difficult to measure low-temperature C

- When measurements went down to low temperatures

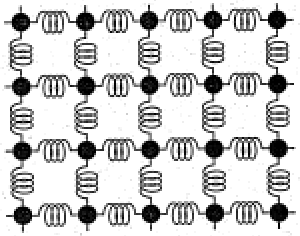


Einstein (1908)

- Energy in solids shows up as vibrations of atoms
- Vibrations ⇒ Oscillator physics
- Recall what Planck's formula says about Oscillator's Energies

Einstein: Each atom vibrates independently with same freq. f_E

$$\Rightarrow \underset{\substack{\uparrow \\ \text{3 directions}}}{3}N \text{ oscillators}$$



$$U(T) = \text{energy of oscillators at temp } T = 3N \cdot \underbrace{\overbrace{\frac{hf_E}{e^{\frac{hf_E}{k_BT}}-1}}^{\text{Planck}}}_{\substack{\text{requires oscillator} \\ \text{to carry discrete} \\ \text{allowed energies}}}$$

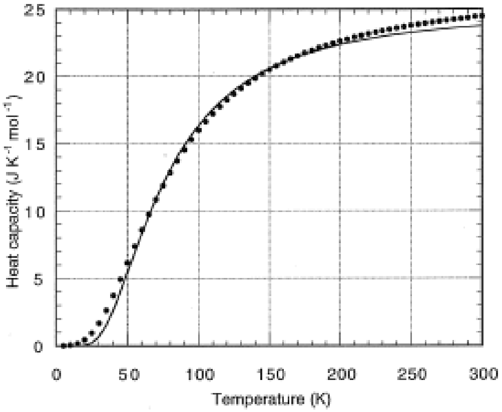
$$C(T) = \frac{dU}{dT} = 3Nk\left(\frac{hf_E}{k_BT}\right)^2 \frac{e^{\frac{hf_E}{k_BT}}}{\left[e^{\frac{hf_E}{k_BT}}-1\right]^2} \quad \text{(Ex.)}$$

- 1 mole $\Rightarrow N = N_A$ = Avogadro's #
- Call $\frac{hf_E}{k_B} \equiv \theta_E$ = "Einstein Temp."

$$\boxed{C_{molar}(T) = 3R\left(\frac{\theta_E}{T}\right)^2 \cdot \frac{e^{\theta_E/T}}{\left[e^{\theta_E/T}-1\right]^2}}$$

"Einstein's model of heat capacity"

$$\underbrace{\underline{\text{It drops to zero as } T \text{ drops!}}}_{\text{1st time a theory shows this behavior}}$$



• Data — Line (Einstein model)

Implications

- Assumed oscillator of freq. f could only have $(0, hf, 2hf, \cdots, nhf, \cdots)$ discrete energies
- Quantum Mechanics applied to oscillator should give these results!

Extensions (Self-learning/Optional)

- As expt's went to $T \approx 0$, Einstein's formula deviates from data!
- Debye model of heat capacity works better! (see solid state physics)
- How about heat capacity of metals (conductors)? (Fermi gas physics)

D. Photoelectric Effect: $E = hf$ (Particle Nature of Light)

- See PHYS1122 notes [Early exp'ts by Lenard (1905 Nobel Prize)]
- Only when $hf > \phi$ (work function of sample), electrons come out with $\boxed{k.e. = hf - \phi}$
- Implication
  - Light: EM Waves (Maxwell's Eqs. ~1870) [$\overset{\text{frequency}}{f}$ & $\overset{\text{wavelength}}{\lambda}$ describe Wave properties]
  - Thermal radiation and photoelectric effect

    Light of freq. f consists of energy packets (photons) of energy

$$\boxed{\underset{\text{particle property}}{E} = h\underset{\text{wave property}}{f}}$$

$\boxed{\text{Higher intensity} \Rightarrow \text{More photons, each carries } E = hf}$

- Millikan (1916) did careful exp'ts and verified Einstein's idea and measured h
- Einstein: 1921 Nobel Prize

  Millikan: 1923 Nobel Prize (for photoelectric effect exp'ts and measuring e)

Extensions / Self-Learning

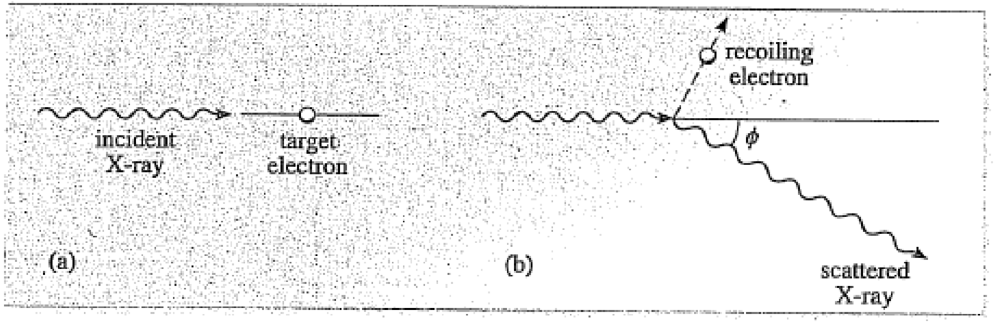
- Lesson to learn: Experimental techniques/phenomena that drived understanding of fundamental physics often developed into new research tools!
- Photoemission Electron Spectroscopy
  - Use emitted electrons to infer properties of sample
    ⇒ tools for studying materials
  - PEEM or PEM: Photoemission Electron Microscopy
  - ARPES: Angle resolved photoemission spectroscopy

  Look up what they are (Ex.)

[1981 Nobel Prize to Siegbahn for "... development of high-resolution electron spectroscopy".]

E. Compton Effect: $E = hf$ and $p = \frac{h}{\lambda}$ (Particle Nature of Light)

- See PHYS1122 notes
- Compton (1919 - 1923) [1927 Nobel Prize]



(a) The Compton effect. An X-ray or γ-ray 'particle' collides with a slow moving electron in one of the target atoms. (b) The electron recoils, absorbing energy from the X-ray particle which is scattered into a new direction and with increased wavelength.

- To explain exp'tal results, a photon has E and p (momentum) given by

$$\boxed{\underset{\text{particle}}{E} = h\underset{\text{wave}}{f} \quad \text{and} \quad \underset{\text{particle}}{p} = \frac{h}{\underset{\text{wave}}{\lambda}}}$$

- a (photon) particle collides with another particle (electron)

$$\boxed{\text{Particle Nature of Light (Wave) as revealed by experiments in first 20 years of 20th century}}$$

Implications

- Let's see how the mathematical form of EM waves looks when we replace f and λ by E and p [we are using standard wave formula here]

$$\vec{E} = \underset{\nearrow\ \text{amplitude}}{\vec{E}_0} \cos(\underset{\uparrow\ \text{wavenumber}}{k}x - \underset{\nwarrow\ \text{angular freq.}}{\omega t})$$ [propagating in x]

$$= \vec{E}_0 \cos\left(\frac{2\pi}{\lambda}x - 2\pi f t\right)$$ [expressed in terms of wave symbols λ and f]

$$= \vec{E}_0 \cos\left(\frac{2\pi}{h}px - \frac{2\pi}{h}Et\right)$$ [expressed in terms of p and E, h enters]

$$= \vec{E}_0 \cos\left(\frac{p}{\hbar}x - \frac{E}{\hbar}t\right)$$ [defined $\hbar = \frac{h}{2\pi}$; $\underset{\nearrow\ \text{particle}}{p} = \frac{h}{\lambda} = \hbar \underset{\uparrow\ \text{wave}}{k}$]

In complex form:

$$\boxed{\vec{E} = \mathrm{Re}\left[\vec{E}_0\, e^{i\frac{p}{\hbar}x}\, \underbrace{e^{-i\frac{E}{\hbar}t}}_{\text{time part}}\right]}$$ [it is still about light]

- In QM, a wavefunction describes the state of a quantum system consisting of a particle. It is in general complex (like [⋯] above). A state of definite energy E evolves as $e^{-iEt/\hbar}$ in time. [⋯] above describes a free particle (matter) in QM with $p^2/2m = E$.

- $E = hf$ and $p = \dfrac{h}{\lambda}$ for light

For light (EM waves) in vacuum,

$$c = \underset{\text{about time}}{f}\,\overset{\text{(appears with } x\text{) about space}}{\lambda} \quad \text{or} \quad c\,\underset{\text{about space}}{k} = \overset{\text{(appears with } t\text{) about time}}{\omega}$$

[connected through dispersion relation, e.g. $ck = \omega$]

Specific to light:

$$E = hf = \frac{hc}{\lambda} = cp \quad \text{or} \quad \boxed{E = cp} \quad \left\{ \text{photon is massless} \right.$$

- The name "photon" was introduced in 1926 by Lewis.

About light, we have:

- Maxwell's Wave theory (great success)
- particle nature

Ready for the most important discussion (two-slit exp't) that reveals the secret of light and Quantum Mechanics

Pay special attention to the next section. It is basically all of QM (without math).